Answers to Coursebook questions – Chapter 5.6

Questions marked with a star (*) use the formula for the magnetic field created by a current ($B = \frac{\mu I}{2\pi r}$) which is not on the syllabus and so is not examinable.

1 See **Figure 6.21** (page 343 in *Physics for the IB Diploma*).

2 We must apply the right-hand rule for force.

a The magnetic field is into the page.

b The force is into the page.

c The magnetic field is out of the page.

d The force is zero since the velocity is anti-parallel to the field.

e The force is zero since the velocity is parallel to the field.

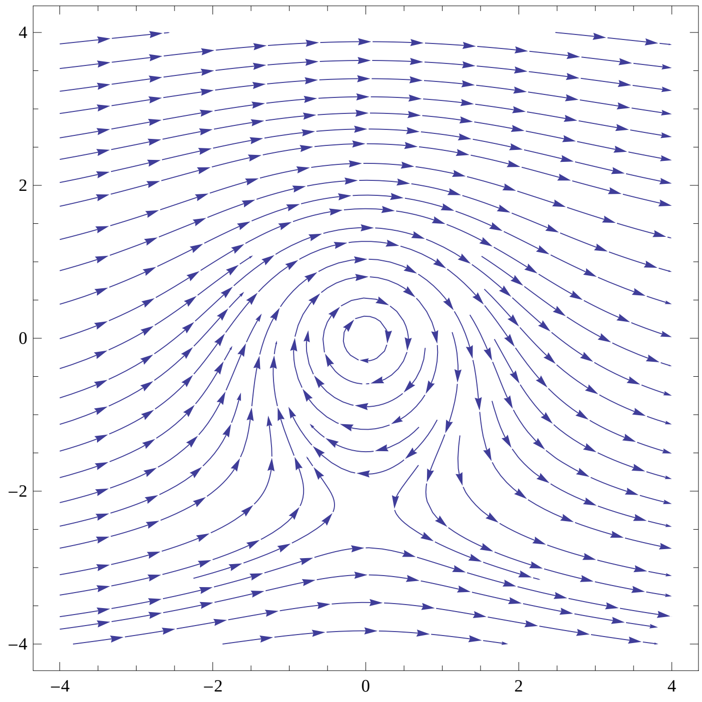
3* The current of 4.0 A produces a magnetic field into the page at P. The other current produces a magnetic field out of the page. The magnitudes are
$\frac{4\pi\times10^{-7}\times4.0}{2\pi\times0.07}=1.14\times10^{-5}\ \text{T}$ and $\frac{4\pi\times10^{-7}\times3.0}{2\pi\times0.10}=6.0\times10^{-6}\ \text{T}$.
So the net field is $1.14\times10^{-5}-6.0\times10^{-6}=5.4\times10^{-6}\ \text{T}$ into the page.

4* At P, the field from the top wire is $\frac{4\pi\times10^{-7}\times5.0}{2\pi\times4.0\times10^{-3}}=2.50\times10^{-4}\ \text{T}$ out of the page and from the bottom wire it is $\frac{4\pi\times10^{-7}\times5.0}{2\pi\times8\times10^{-3}}=1.25\times10^{-4}\ \text{T}$ also out of the page.
The net field at P is $3.75\times10^{-4}\ \text{T}$ out of the page.

At Q, the field is zero since the wires produce magnetic fields of equal magnitude and opposite direction.

At R, the top wire produces a field $\frac{4\pi\times10^{-7}\times5.0}{2\pi\times10\times10^{-3}}=1.00\times10^{-4}\ \text{T}$ into the page and the bottom wire produces a field $\frac{4\pi\times10^{-7}\times5.0}{2\pi\times6.0\times10^{-3}}=1.67\times10^{-4}\ \text{T}$ into the page.
The net field is $2.67\times10^{-4}\ \text{T}$ into the page.

5



6 The magnetic field is directed into the page. In **a** the right-hand rule (for a negative charge) gives a force downwards away from the wire. In **b** it gives a force to the right.

7 In **a** the field is to the right and so the force is into the page. In **b** the velocity is parallel to the field and the force is zero. In **c** the force is towards the magnet (up the page).

8 No, because the field inside the solenoid is along the axis and so parallel to the velocity of the electron.

9 We apply the right-hand rule for force in each case to find: field out of page, field out of page, field to the left, field to the left.

10* **a** The 2.0 A current attracts the current in DC and repels the current in BA. The forces on BC and DA cancel out. The magnetic field created at DC is $\frac{4\pi\times10^{-7}\times2.0}{2\pi\times5.0\times10^{-2}}=8.0\times10^{-6}\text{ T}$ and at BA it is $\frac{4\pi\times10^{-7}\times2.0}{2\pi\times10\times10^{-2}}=4.0\times10^{-6}\text{ T}$.
So the net force on the loop is upwards and has magnitude
$B_1IL-B_2IL=(4.0\times10^{-6}-4.0\times10^{-6})\times0.50\times0.15=3.0\times10^{-7}\text{ N}$.

b If the current in the loop is reversed the magnitude of the force will stay the same but the force will reverse direction.

11* **a** By the right-hand rule for force, the force on AB is into the page, on BC it is zero (current and field parallel), on CD it is out of the page and zero on DA. The magnitude of the force on AB and CD is $BIL=0.050\times2.0\times0.20=0.020\text{ N}$.

b The net force is zero.

12* From $B=\frac{\mu_0 NI}{l}$ we find $2.26\times10^{-3}=\frac{4\pi\times10^{-7}\times15\times N}{0.30}\Rightarrow N\approx 36$.
Hence the length of wire needed is $L=2\pi RN=2\pi\times0.12\times36\approx 27\text{ m}$.

13 By the right-hand rule, at P the field is out of the page and at Q it is into the page.

14* **a** The magnetic field can only be zero along a line in between the wires and on the same plane as the wires. This is because the field from the top wire is into the page, whereas that of the bottom wire is out of the page. Let the distance from the top wire be x. Then $\frac{4\pi\times10^{-7}\times2.0}{2\pi\times x}=\frac{4\pi\times10^{-7}\times3.0}{2\pi\times(20-x)}$ giving $\frac{2.0}{x}=\frac{3.0}{(20-x)}$ or $40-2x=3x\Rightarrow x=8.0\text{ cm}$.

b Now the fields are opposite above the top wire or below the bottom wire. Since the bottom current is the largest, the place where the field is zero has to be above the top wire. Let the distance from the top wire be x. Then $\frac{4\pi\times10^{-7}\times2.0}{2\pi\times x}=\frac{4\pi\times10^{-7}\times3.0}{2\pi\times(20+x)}$ giving $\frac{2.0}{x}=\frac{3.0}{(20+x)}$ or $40+2x=3x\Rightarrow x=40\text{ cm}$.

15 **a** The electric and magnetic forces on the electron must be equal and opposite. The electric force is directed upwards and so the magnetic force is downwards. Hence, by the right-hand rule for force, the magnetic field must be directed into the page.
The electric field is $E=\frac{V}{d}=\frac{120}{5.0\times10^{-2}}=2.4\times10^{3}\text{ N C}^{-1}$.

To find the magnitude, use $qE=qvB\Rightarrow B=\frac{E}{v}=\frac{2.4\times10^{3}}{2.0\times10^{6}}=1.2\times10^{-3}\text{ T}$.

b The electric force and the magnetic force would change direction but keep the same magnitude. So the net force will be zero for a proton with the same speed as that of the electron in **a**.

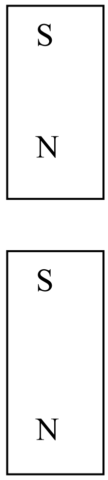
c If the speed is doubled the magnetic force would double but the electric force would be unchanged. Hence the forces will no longer balance and there will be a deflection downwards.

16 **a** There would be equal and opposite forces at the ends of the bar magnet giving a net force of zero.

b The forces would, however, force the bar magnet to rotate.

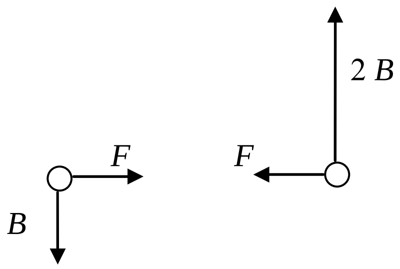
17 The force would be $F=BIL\sin\theta=5.0\times10^{-5}\times3.0\times10^{3}\times30\times\sin30^{\circ}=2.25\text{ N}$.

18 Since the current is counter-clockwise in both cases, the loops create magnetic fields that can be approximated by small bar magnets as shown below.



Hence they will attract.

19



20 The radius of the circular path is found from $qvB = \dfrac{mv^2}{R} \Rightarrow R = \dfrac{mv}{qB}$.

The electron takes a time $\dfrac{2\pi R}{v} = \dfrac{2\pi}{v}\dfrac{mv}{qB} = \dfrac{2\pi m}{qB}$ to complete a revolution and so the number of revolutions per second is $f = \dfrac{qB}{2\pi m}$. A proton has a larger mass and so would complete a smaller number of revolutions per second.

21 **a** The initial velocity of the proton may be decomposed into a component parallel to the positive *x*- axis and a component parallel to the positive *z*-axis. The components are $v_x = v\cos\theta = 1.5\times10^6 \times \cos 30^\circ = 1.30\times10^6\ \text{m s}^{-1}$ and $v_z = v\sin\theta = 1.5\times10^6 \times \sin 30^\circ = 0.75\times10^6\ \text{m s}^{-1}$. Since the component along the *z*-axis is parallel to the magnetic field there will be no force in the vertical direction. The *x*-component of velocity is at right angles to the magnetic field and so the proton will experience a force $F = qv_xB$ which will make the proton to perform a circle parallel to the *x*–*y* plane with speed v_x just as it also moves upwards with constant speed v_z. This means that the path of the proton is a helix (a spiral).

b The radius is found from $qv_x B = \frac{mv_x^2}{R} \Rightarrow R = \frac{mv_x}{qB}$.

Numerically this is $R = \frac{1.67\times10^{-27}\times1.30\times10^{6}}{1.6\times10^{-19}\times0.50} = 0.027\text{ m}$.

c The proton takes a time $\frac{2\pi R}{v_x} = \frac{2\pi}{v_x}\frac{mv_x}{qB} = \frac{2\pi m}{qB}$ to complete a revolution and so the number of revolutions per second is $f = \frac{qB}{2\pi m} = \frac{1.6\times10^{-19}\times0.50}{2\pi\times1.67\times10^{-27}} = 7.6\times10^{6}$.

d $v_z = 0.75\times10^{6}\text{ m s}^{-1}$.

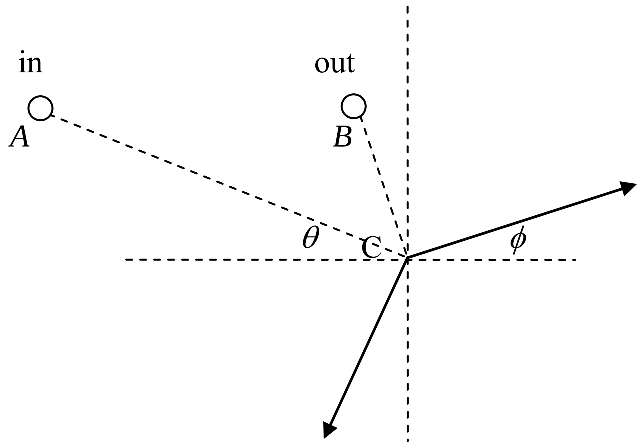
e The period (time for one revolution) is the inverse of the frequency and so $T = \frac{1}{7.6\times10^{6}} = 1.3\times10^{-7}\text{ s}$ and so the distance covered vertically in this time is $z = v_z T = 0.75\times10^{6}\times1.3\times10^{-7} = 9.8\times10^{-2}\text{ m}$.

22 A charged particle performs a circle inside a magnetic field with period $T = \frac{2\pi m}{qB}$.

In other words the particle sweeps an angle of 360° in a time equal to the period.
In this case the particle only sweeps an angle of 30° and so this will take a time
$\frac{1}{12}\times T = \frac{2\pi m}{12qB} = \frac{\pi m}{6qB} = \frac{\pi\times9.1\times10^{-31}}{6\times1.6\times10^{-19}\times0.50} = 5.96\times10^{-12}\text{ s}$.

23* The 4.0 A current creates a field downwards at P. The other two create a field upwards.
The fields are:
$B_1 = \frac{4\pi\times10^{-7}\times4.0}{2\pi\times15\times10^{-2}} = 5.3\times10^{-6}\text{ T}$, $B_2 = \frac{4\pi\times10^{-7}\times2.0}{2\pi\times10\times10^{-2}} = 4.0\times10^{-6}\text{ T}$ and
$B_3 = \frac{4\pi\times10^{-7}\times1.0}{2\pi\times5.0\times10^{-2}} = 4.0\times10^{-6}\text{ T}$.
The net field is then $B_2 + B_3 - B_1 = 2.7\times10^{-6}\text{ T}$ upwards.

24* A diagram is the following:



Using the cosine rule, the angles of the triangle formed by the currents and the origin of the axes are:

$$A = \cos^{-1}\frac{b^2+c^2-a^2}{2bc} = \cos^{-1}\frac{5.0^2+8.0^2-4.0^2}{2\times 5.0\times 8.0} = 24.1^\circ,$$
$$B = \cos^{-1}\frac{a^2+c^2-b^2}{2ac} = \cos^{-1}\frac{5.0^2+4.0^2-8.0^2}{2\times 5.0\times 4.0} = 125^\circ.$$

Hence the angles in the diagram are $\theta = 24.1^\circ$ and $\phi = 125^\circ - 90^\circ = 35^\circ$.

The field from the 'in' current is $B_1 = \dfrac{4\pi\times 10^{-7}\times 12.0}{2\pi\times 8.0\times 10^{-2}} = 3.0\times 10^{-5}$ T and that from the 'out' current is $B_2 = \dfrac{4\pi\times 10^{-7}\times 10.0}{2\pi\times 4.0\times 10^{-2}} = 5.0\times 10^{-5}$ T.

Therefore, the components of the fields are:

$$B_{1x} = -3.0\times 10^{-5}\times\cos(90^\circ - 24.1^\circ) = -1.22\times 10^{-5}\text{ T and}$$
$$B_{1y} = -3.0\times 10^{-5}\times\sin(90^\circ - 24.1^\circ) = -2.74\times 10^{-5}\text{ T},$$

$B_{2x} = 5.0\times 10^{-5}\times\cos 35^\circ = 4.10\times 10^{-5}$ T and $B_{2y} = 5.0\times 10^{-5}\times\sin 35^\circ = 2.87\times 10^{-5}$ T.

Hence the net field has components $B_x = -1.22\times 10^{-5} + 4.10\times 10^{-5} = 2.88\times 10^{-5}$ T and $B_y = -2.74\times 10^{-5} + 2.88\times 10^{-5} = 0.14\times 10^{-5}$ T. So the net field is

$\sqrt{2.88^2+0.14^2}\times 10^{-5} = 2.9\times 10^{-5}$ T at $\theta = \arctan\dfrac{0.14\times 10^{-5}}{2.88\times 10^{-5}} = 2.8^\circ$ to the horizontal.

25* The magnetic field of wire 1 at wire 2 is $B_1 = \frac{4\pi\times10^{-7}\times3.0}{2\pi\times3.0\times10^{-2}} = 2.0\times10^{-5}\ \text{T}$ directed to the left and the field from wire 3 at wire 2 is $B_3 = \frac{4\pi\times10^{-7}\times4.0}{2\pi\times4.0\times10^{-2}} = 2.0\times10^{-5}\ \text{T}$ directed upwards. The net magnetic field has magnitude $\sqrt{B_1^2+B_2^2} = 2.8\times10^{-5}\ \text{T}$ at 135° to the positive *x*-axis. The force per unit length is then $\frac{F}{L} = BI = 2.8\times10^{-5}\ \text{N} = 28\ \mu\text{N}$.

26 The electron completes a full revolution in a time of $T = \frac{2\pi r}{v}$.

Thus a charge e moves past the circle in this time and so the current created by the revolving electron is $I = \frac{Q}{T} = \frac{\mathrm{e}}{\frac{2\pi r}{v}} = \frac{\mathrm{e}v}{2\pi r}$.

Hence the field created by the electron at the nucleus is $B = \mu_0 \frac{\mathrm{e}v}{4\pi r^2}$.

27*

This is a flawed question as it stands.

The field inside a solenoid is given by $B = \frac{\mu_0 NI}{l}$. The current is found from $I = \frac{V}{R}$ where R is the resistance of the wire used. This resistance is $R = \rho\frac{L}{A}$ where L is the length of the wire. So $B = \frac{\mu_0 NVA}{l\rho L}$. Now the length of the solenoid is $l = N(2r)$ where r is the radius of the wire. Hence $B = \frac{\mu_0 NVA}{N2r\rho L} = \frac{\mu_0 V\pi r^2}{2r\rho L} = \frac{\mu_0 V\pi r}{2\rho L} = \left(\frac{\mu_0 V\pi}{2\rho}\right)\frac{r}{L}$. We have a fixed mass and hence volume of metal. The volume is given by $v = \pi r^2 L$. Substituting in the formula for the magnetic field we get

$$B = \left(\frac{\mu_0 V\pi}{2\rho}\right)\frac{r}{\frac{v}{\pi r^2}} = \left(\frac{\mu_0 V\pi^2}{2\rho v}\right)r^3.$$

So to maximize this field we need a large wire radius. This means that we should use all the metal into a single turn of wire. This is why the question is flawed: with one single turn we do not have a solenoid! There must be additional restrictions placed on this problem such as a given minimum number of turns or solenoid radius.

28* **a** Both currents produce a magnetic field in the same direction and so the net field is just $B = \mu_0 \frac{I}{2\pi(r-\frac{d}{2})} + \mu_0 \frac{I}{2\pi(r+\frac{d}{2})}$.

b We may rewrite

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{(r-\frac{d}{2})} + \frac{1}{(r+\frac{d}{2})}\right)$$

$$= \frac{\mu_0 I}{2\pi r}\left(\frac{1}{(1-\frac{d}{2r})} + \frac{1}{(1+\frac{d}{2r})}\right)$$

$$\approx \frac{\mu_0 I}{2\pi r}\left((1+\frac{d}{2r}) + (1-\frac{d}{2r}) + \ldots\right) \qquad \text{used } \frac{1}{1+x} \approx 1-x$$

$$\approx \frac{\mu_0 2I}{2\pi r} + \ldots$$

This is completely expected. From a large distance away (i.e. $r >> d$) we see just one current of magnitude $2I$ and so the field is expected to be $B = \frac{\mu_0 2I}{2\pi r}$.

c The fields are now opposite and so $B = \mu_0 \frac{I}{2\pi(r-\frac{d}{2})} - \mu_0 \frac{I}{2\pi(r+\frac{d}{2})}$.

d We may rewrite

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{(r-\frac{d}{2})} - \frac{1}{(r+\frac{d}{2})}\right)$$

$$= \frac{\mu_0 I}{2\pi r}\left(\frac{1}{(1-\frac{d}{2r})} - \frac{1}{(1+\frac{d}{2r})}\right)$$

$$\approx \frac{\mu_0 I}{2\pi r}\left((1+\frac{d}{2r}) - (1-\frac{d}{2r}) + \ldots\right) \qquad \text{used } \frac{1}{1+x} \approx 1-x$$

$$\approx \frac{\mu_0 I d}{2\pi r^2} + \ldots$$

In this case, from a large distance away we see zero current to first approximation and so we do not see the familiar $\frac{1}{r}$ in the magnetic field.

29 a The radius of a circular path in a magnetic field is $R = \frac{mv}{qB}$ and so the radius increases when the speed increases.

b Acceleration takes place every time the particle crosses the gap. At that instant, the potential on the other side has to be of the right sign so as to accelerate the particle. After half a revolution the particle will again be at the gap and again it must face a potential on the opposite side that is again of the right sign for acceleration. This sign must be opposite to what it was half a revolution before. Hence the frequency of the alternating source must be equal to the frequency of rotation.

c We know that $R = \dfrac{mv}{qB}$ and so $T = \dfrac{2\pi R}{v} = \dfrac{2\pi m}{qB}$.

d For an electron, $f = \dfrac{1}{T} = \dfrac{qB}{2\pi m} = \dfrac{1.6\times10^{-19}\times0.50}{2\pi\times9.1\times10^{-31}} = 1.4\times10^{10}$.

e The kinetic energy increases by 120 keV at every crossing.
After 100 revolutions there have been 200 crossings and so the energy increased by $200\times120 = 2.4\times10^{4}$ keV $= 24$ MeV $= 3.8\times10^{-12}$ J.

30* The magnetic field at the position of the 100 A wire due to the other wire is $\dfrac{4\pi\times10^{-7}\times200}{2\pi\times0.050} = 8.0\times10^{-4}$ T and is directed downwards. Hence the net field at the position of the 100 A wire is $8.0\times10^{-4} - 3.0\times10^{-4} = 5.0\times10^{-4}$ T downwards.
Hence the force per unit length is $\dfrac{F}{L} = BI = 5.0\times10^{-4}\times100 = 5.0\times10^{-2}$ N m^{-1}.
By the right-hand rule for force, the force is directed to the left.

31 **a** The combined magnetic field from the two wires at point R must point downwards so as to cancel the uniform field.
Since R is closer to Q, the field of Q is larger than the field from P.
Hence the current in Q must go out of the page.

b If the current increases, the net field from P and Q increases as well, so that the total field at R is no longer zero. If we move closer to Q the field from Q will be much larger than the field from P and so their combined field will be downwards and much larger than external field. Hence the point has to move to the left.

32 The radius of the circular path is given by $R = \dfrac{mv}{qB}$ and so the period is $T = \dfrac{2\pi R}{v} = \dfrac{2\pi m}{qB}$. Since the particles are identical,

a $\dfrac{T_2}{T_1} = 1$ and **b** $\dfrac{E_2}{E_1} = \dfrac{\frac{1}{2}mv_2^2}{\frac{1}{2}mv_1^2} = \dfrac{v_2^2}{v_1^2} = 2^2 = 4$.